Semiconductors Physics:

Syllabus

- **Basic Electronics**
  - It is evident that many years of research by a great many people, both before and after the discovery of the transistor effect, has been required to bring our knowledge of semiconductors to its present development. We were fortunate to be involved at a particularly opportune time and to add another small step to the control of nature for the benefit of mankind.
  - John Bardeen, 1956 Nobel lecture

- **Basic Electronics**
  - **Syllabus**
    - **Basics of Semiconductors**:
      - Crystal Structure, Energy Bands in solids, Classification of solids, classification of materials, Electron emission from solids, classification of semiconductors, properties of semiconductors, Hall effect, Diffusion, thermistors and photoconductors, Advantages of semiconductor devices.

- **Syllabus**
  - **Diodes and Transistors**:
    - Semiconductor junction diodes, diode circuits, junction transistors, characteristics, Biasing and amplification, Properties of CE, CB and CE amplifiers, high Frequency effects in transistors, noise in Transistors, Basic voltage and power amplifiers, feedback in amplifiers.

- **Syllabus**
  - **Field Effect transistors**:
    - Junction field effect transistors (JFET), static characteristics of JFET, pinch off voltage, Metal oxide Semiconductors FET (MOSFET), static characteristics of MOSFET, Biasing of FET, FET parameters.

- **Syllabus**
  - **IC and OP AMP**:
    - Fabrication of monolithic IC, integration of circuit components, Limitations of ICs, Advantages of IC’s over analog circuits, Operational Amplifier (OP AMP), Basic operations of OP AMP, Applications of OP AMP.

- **Syllabus**
  - **Introduction to Digital Systems**:
    - Flip-Flop, Registers, Counters, K-map
  - **Intermodulation**:
    - Modulation and Demodulation
Reference Books:
- Basic Electronics by B. Basavaraj and H N Shivashanker
- Physics of Semiconductor Devices by S. M. Sze, John Wiley & Sons Publications
- Electronics: Analog and Digital by I. J. Nagrath, PHI publications
- Electronic Devices and Circuits by Allen Mottershead, PHI publications

Course Plan:

<table>
<thead>
<tr>
<th>UNIT</th>
<th>Lecture + Tutorial (in hours)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basics of Semiconductors</td>
<td>10</td>
</tr>
<tr>
<td>Diodes and Transistors</td>
<td>9</td>
</tr>
<tr>
<td>Field Effect Transistors</td>
<td>7</td>
</tr>
<tr>
<td>Modulation and Demodulation</td>
<td>7</td>
</tr>
<tr>
<td>IC and OP AMP</td>
<td>9</td>
</tr>
<tr>
<td>Total</td>
<td>42</td>
</tr>
</tbody>
</table>

Evaluation

<table>
<thead>
<tr>
<th>Component</th>
<th>Duration</th>
<th>Weightage (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minor Test 1</td>
<td>One Hour</td>
<td>20</td>
</tr>
<tr>
<td>Minor Test 2</td>
<td>One Hour</td>
<td>20</td>
</tr>
<tr>
<td>Basic Electronics Lab</td>
<td></td>
<td>20</td>
</tr>
<tr>
<td>Major Test</td>
<td>Two Hour</td>
<td>40</td>
</tr>
</tbody>
</table>

Laboratory

<table>
<thead>
<tr>
<th>Category</th>
<th>Cost Centre</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basic Electronics Lab</td>
<td>PHI publications</td>
</tr>
<tr>
<td>Major Test</td>
<td>Tata McGraw Hills Publications</td>
</tr>
</tbody>
</table>

Categories of Solids

- There are three categories of solids, based on their conducting properties:
  - conductors
  - semiconductors
  - insulators
Reviewing the previous table reveals that:

- The electrical conductivity at room temperature is quite different for each of these three kinds of solids
- **Metals and alloys** have the highest conductivities
- followed by **semiconductors**
- and then by **insulators**

**Semiconductor Conduction**

- The **free-electron** model from Physics does not apply to semiconductors and insulators, since these materials simply lack enough free electrons to conduct in a free-electron mode.

There is a different conduction mechanism for semiconductors than for normal conductors.

**Semiconductors** are a group of materials having conductivities between those of metals and insulators.

Two general classifications of semiconductors are elemental semiconductor materials (e.g., Si and Ge), compound semiconductor materials (e.g., AlP, AlAs, GaP, GaAs, InP) and Diluted magnetic Semiconductors.

Silicon is by far the most common semiconductor used in integrated circuits.

Gallium arsenide is the most common of compound semiconductors. Its good optical properties make it useful in optical devices. It is also used in specialized applications, for example, when high speed is required.

There are more complex compound semiconductors, for example, Al,Ga,As, which provide flexibility when choosing material properties.
Types of Solids

There are three general types of solids: amorphous, polycrystalline, and single crystal. They are characterized by the size of ordered regions within the material. An ordered region is a spatial volume in which atoms or molecules have a regular geometric arrangement or periodicity.

Amorphous materials: order within a few atomic or molecular dimensions. Polycrystalline materials: a number of regions with order over many atomic or molecular dimensions. These ordered regions are called grains, which are separated from one another by grain boundaries. Single crystal materials: a high degree of order or regular geometric periodicity throughout the entire volume of the material.

Lattices and Unit Cells

A representative unit, or group of atoms in single crystal materials is repeated at regular intervals in each of the three dimensions. The periodic arrangement of the unit or group in the crystal is called the lattice. Each unit or group can be represented by a dot, which is called a lattice point.

Single crystal lattices have translational symmetries (assuming each crystal is infinitely large in space). They can be characterized by three noncollinear directions. These translation directions need not be perpendicular.

A Generalized Primitive Cell

The lattice is characterized by three vectors $a$, $b$, and $c$, which need not be perpendicular and which may or may not be equal in length. Every equivalent lattice point in the three-dimensional crystal can be found using the vector

$$ r = pa + qb + sc $$

where $p$, $q$, and $s$ are integers.

Examples of Some Basic Crystal Structures

For these structures, we can choose three vectors $a$, $b$, and $c$ that are perpendicular to each other and equal in length. The sc structure has an atom located at each corner. The bcc structure has additional atom at the center of the cube. The fcc structure has additional atoms on each face plane.
Crystal Planes and Miller Indices

Real crystals are not infinitely large. They have surfaces. Crystal surfaces are often related to different lattice planes. Lattice planes are characterized with Miller indices \( \{hkl\} \), which are a set of integers with no common factors, inversely proportional to the intercepts of the crystal plane along the crystal axes:

\[
P = \frac{h}{a} \frac{k}{b} \frac{l}{c}
\]

The reciprocals of the intercepts are

\[
\frac{1}{p}, \frac{1}{q}, \frac{1}{s}
\]

Multiply the smallest common denominator, which is 6 in this case, to obtain

\[
\{1 \ 1 \ 1\}\text{ or } \{2 \ 3 \ 6\}
\]

Parallel lattice planes have same Miller indices and are entirely equivalent to each other.

Surface Density of Atoms on a Particular Plane

**Example:** consider the bcc structure. Assume the atoms can be represented as hard spheres with the closest atoms touching each other and the lattice constant is 0.5 nm. Calculate the surface density of atoms on the \((110)\) plane.

Consider the atoms on the \((110)\) plane. The atom at each corner is shared by four similar rectangles. So one fourth of each atom at the corner contributes to the shaded rectangle. The four corner atoms effectively contribute one atom to the shaded rectangle. The atom at the center is not shared by any other rectangle. It is entirely included in the shaded rectangle. Therefore, the shaded rectangle contains two atoms.

Surface Density of Atoms on a Particular Plane

**Solution:** the surface density is found by dividing the number of lattice atoms by the surface area.

\[
\text{Surface density} = \frac{2}{(8 \times 0.5 \sqrt{2})} = \frac{2}{5 \times 10^{-4} \sqrt{2}} = 5.66 \times 10^{14} \text{ atoms per cm}^2
\]

The surface density of atoms is a function of the particular crystal plane and generally varies from one crystal plane to another.

Description of directions in a lattice:

In addition to lattice planes, we also want to describe a particular direction in the crystal. The direction can be expressed as a set of three integers that are the components of a vector in that direction. For example, the body diagonal in the sc lattice has vector components of 1,1,1. The body diagonal is then described as the \([111]\) direction. The brackets are used to designate direction as distinct from the parentheses used for crystal planes.

The Diamond Structure

Silicon is the most common semiconductor material. Both silicon and germanium have a diamond crystal structure.

The basic building block of the diamond structure is the tetrahedral structure.

An important characteristic of the diamond structure is that any atom in the structure has four nearest neighboring atoms. All atoms in the diamond structure are of the same species, such as silicon or germanium.
The zincblende (sphalerite) structure differs from the diamond structure only in that there are two different types of atoms in the structure. Compound semiconductors, such as GaAs, have the zincblende structure. The important feature of both the diamond and zincblende structure is that the atoms are joined together to form tetrahedrons.

### Classification of lattice

#### The Seven Crystal System

And

#### The Fourteen Bravais Lattices

<table>
<thead>
<tr>
<th>System</th>
<th>Unit Cell Shape</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Cubic</td>
<td>(a=b=c, \alpha=\beta=\gamma=90^\circ)</td>
</tr>
<tr>
<td>2. Tetragonal</td>
<td>(a=b\neq c, \alpha=-\beta=-\gamma=90^\circ)</td>
</tr>
<tr>
<td>3. Orthorhombic</td>
<td>(a\neq b\neq c, \alpha=\beta=-\gamma=90^\circ)</td>
</tr>
<tr>
<td>4. Hexagonal</td>
<td>(a=b\neq c, \alpha=\beta=90^\circ, \gamma=120^\circ)</td>
</tr>
<tr>
<td>5. Rhombohedral</td>
<td>(a=b=c, \alpha=\beta=-\gamma=90^\circ)</td>
</tr>
<tr>
<td>6. Monoclinic</td>
<td>(a=b\neq c, \alpha=90^\circ \neq \gamma)</td>
</tr>
<tr>
<td>7. Triclinic</td>
<td>(a=b\neq c, \alpha\neq \beta\neq \gamma)</td>
</tr>
</tbody>
</table>
14 Bravais lattices divided into seven crystal systems

Crystal system
1. Cubic
2. Tetragonal
3. Orthorhombic
4. Hexagonal
5. Trigonal
6. Monoclinic
7. Triclinic

Bravais lattices
P I F C
Semiconductors Physics:

Face centered cubic in the Bravais list?

**Cubic F = Tetragonal I**

14 Bravais lattices divided into seven crystal systems

<table>
<thead>
<tr>
<th>Crystal system</th>
<th>Bravais lattices</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cubic</td>
<td>P</td>
</tr>
<tr>
<td>Tetragonal</td>
<td>I</td>
</tr>
<tr>
<td>Orthorhombic</td>
<td>F</td>
</tr>
<tr>
<td>Hexagonal</td>
<td>P</td>
</tr>
<tr>
<td>Trigonal</td>
<td>P</td>
</tr>
<tr>
<td>Monoclinic</td>
<td>C</td>
</tr>
<tr>
<td>Triclinic</td>
<td>P</td>
</tr>
</tbody>
</table>

Notation

- **P**: Primitive (lattice points only at the corners of the unit cell)
- **I**: Body-centred (lattice points at the corners + one lattice point at the centre of the unit cell)
- **F**: Face-centred (lattice points at the corners + lattice points at centers of all faces of the unit cell)
- **C**: End-centred or base-centred (lattice points at the corners + two lattice points at the centers of a pair of opposite faces)

What is the basis for classification of lattices into 7 crystal systems and 14 Bravais lattices?

Lattices are classified on the basis of their symmetry.

ML Frankenheim
Auguste Bravais
1811-1863

Couldn’t find his photo
Your photo

15 lattices
14 lattices
13 lattices
Semiconductors Physics:

What is symmetry?

If an object is brought into self-coincidence after some operation it said to possess symmetry with respect to that operation.

Rotational symmetry

A rectangle comes into self-coincidence by 180 degrees rotation

Rotation Axis

If an object come into self-coincidence through smallest non-zero rotation angle of $\theta$ then it is said to have an n-fold rotation axis where

$$n = \frac{360^\circ}{\theta}$$

- $n=180^\circ$, $n=2$, 2-fold rotation axis
- $n=90^\circ$, $n=4$, 4-fold rotation axis

Reflection (or mirror symmetry)

Translational symmetry

Lattices also have translational symmetry
Symmetry classification of lattices

Based on rotational and reflection symmetry alone
⇒ 7 types of lattices
⇒ 7 crystal systems

Based on complete symmetry, i.e., rotational, reflection and translational symmetry
⇒ 14 types of lattices
⇒ 14 Bravais lattices

7 crystal Systems

<table>
<thead>
<tr>
<th>System</th>
<th>Required symmetry</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Cubic</td>
<td>Three 4-fold axis</td>
</tr>
<tr>
<td>2. Tetragonal</td>
<td>one 4-fold axis</td>
</tr>
<tr>
<td>3. Orthorhombic</td>
<td>three 2-fold axis</td>
</tr>
<tr>
<td>4. Hexagonal</td>
<td>one 6-fold axis</td>
</tr>
<tr>
<td>5. Rhombohedral</td>
<td>one 3-fold axis</td>
</tr>
<tr>
<td>6. Monoclinic</td>
<td>one 2-fold axis</td>
</tr>
<tr>
<td>7. Triclinic</td>
<td>none</td>
</tr>
</tbody>
</table>

Atomic Bonding

The formation of one particular crystal structure depends on the interaction or the type of bond, between atoms in the crystal. The interaction between atoms can be described by quantum mechanics. The interaction of atoms tends to form closed valence shells. The elements (e.g., Na, K) in group I tend to lose their one electron and become positively charged. The elements (e.g., Cl, Br) in group VII tend to gain one electron and become negatively charged. These oppositely charged ions experience a Coulomb attraction to form a bond referred to as an ionic bond. Some examples of ionic bonding are NaCl and KBr.

Another atomic bond that tends to achieve closed valence shells is covalent bonding. For example, in the hydrogen molecule, a hydrogen atom has one electron and needs one more electron to complete the valence shell. Covalent bonding results in electrons being shared between atoms so that in effect the valence shell of each atom is full.

Si has four valence electrons. Each Si atom needs four more electrons to complete the valence shell. Therefore, each Si atom forms four covalent bonds with four neighboring Si atoms, which results in the formation of diamond structure. The third major type of atomic bonding is referred to as metallic bonding. Group I elements have one valence electron. They tend to lose one electron to become positively charged. For example, when a large number of Na atoms are brought into close proximity.

Another type of bonding is the Van der Waals bond, which is the weakest of atomic bonds. It results from weak dipole interactions between molecules. Solids formed by this type of bond have low melting temperatures.
Semiconductors Physics:

Covalent Bonding

(a) A tetrahedron bond.  
(b) Schematic two-dimensional representation of a tetrahedron bond.

Conduction of electrons and holes

(a) A broken bond at Position A, resulting in a conduction electron and a hole.  
(b) A broken bond at position B.

Imperfections in Solids

In real crystals, lattices are not perfect. They contain imperfections and impurities, which can alter the electrical properties of solid materials.

One type of imperfection that all crystals have in common is called lattice vibration, which is induced by the thermal energy. The thermal motion causes the distance between atoms to randomly fluctuate, slightly disrupting the perfect geometric arrangement of atoms. Another type of defect is called point defect, including vacancy and interstitial. Vacancy and interstitial not only break the perfect geometric arrangement, but also disrupt the ideal chemical bonding between atoms. A vacancy and interstitial may be in close enough proximity to exhibit an interaction between the two point defects, which is known as a Frenkel defect.

A line defect occurs when an entire row of atoms is missing from its normal site. This defect is referred to as a line dislocation. As with a point defect, a line dislocation disrupts both the normal geometric periodicity of the lattice and the ideal atomic bonds in the crystal.

Impurities in Solids

Impurity atoms may be present in a crystal. Impurity atoms may be located at normal lattice sites, in which case they are called substitutional impurities. Impurity atoms may also be located between normal lattice sites, in which case they are called interstitial impurities. Controlled amounts of particular impurity atoms can favorably alter the electrical properties of semiconductors. The technique of introducing impurity atoms into a semiconductor in order to alter its conductivity is called doping. Two general methods of doping are impurity diffusion and ion implantation.

Band Theory of Solids

- In order to account for decreasing resistivity with increasing temperature as well as other properties of semiconductors, a new theory known as the band theory is introduced.
- The essential feature of the band theory is that the allowed energy states for electrons are nearly continuous over certain ranges, called energy bands, with forbidden energy gaps between the bands.
Band Theory of Solids

- Consider initially the known wave functions of two hydrogen atoms far enough apart so that they do not interact.

![Graph showing wave functions for symmetric and antisymmetric states.]

- An atom in the symmetric state has a nonzero probability of being halfway between the two atoms, while an electron in the antisymmetric state has a zero probability of being at that location.

Kronig-Penney Model

- An effective way to understand the energy gap in semiconductors is to model the interaction between the electrons and the lattice of atoms.

- R. de L. Kronig and W. G. Penney developed a useful one-dimensional model of the electron lattice interaction in 1931.

Kronig-Penney Model

- Kronig and Penney assumed that an electron experiences an infinite one-dimensional array of finite potential wells.

- Each potential well models attraction to an atom in the lattice, so the size of the wells must correspond roughly to the lattice spacing.

\[ \psi = Ae^{ikx} + Be^{-ikx} \]

where the wave number \( k \) is given by the usual relation:

\[ k^2 = \frac{2mE}{\hbar^2} \]
Tunneling

- In the region between \( a < x < a + b \) the electron can tunnel through and the wave function loses its oscillatory solution and becomes exponential:

\[
\psi = C e^{kx} + D e^{-kx}
\]

\[
k^2 = 2m(V_0 - E) / \hbar^2
\]

Kronig-Penney Model

- Matching solutions at the boundary, Kronig and Penney find

\[
\frac{k^2 b}{2} \sin(ka) + \cos(ka) = \cos(Ka)
\]

Here \( K \) is another wave number.

Kronig-Penney Model

- The left-hand side is limited to values between +1 and -1 for all values of \( K \).
- Plotting this it is observed there exist restricted (shaded) forbidden zones for solutions.

The Forbidden Zones

Figure 11.5 (a) Plot of the left side of Equation (11.3) versus \( ka \) for \( \kappa^2 ba / 2 = \frac{3 \pi}{2} \). Allowed energy values must correspond to the values of \( k \) for which the plotted function lies between -1 and +1. Forbidden values are shaded in light blue. (b) The corresponding plot of energy versus \( ka \) for \( \kappa^2 ba / 2 = \frac{3 \pi}{2} \), showing the forbidden energy zones (gaps).

Important differences between the Kronig-Penney model and the single potential well

1) For an infinite lattice the allowed energies within each band are continuous rather than discrete. In a real crystal the lattice is not infinite, but even if chains are thousands of atoms long, the allowed energies are nearly continuous.

2) In a real three-dimensional crystal it is appropriate to speak of a wave vector \( \mathbf{k} \). The allowed ranges for \( \mathbf{k} \) constitute what are referred to in solid state theory as Brillouin zones.

And…

3) In a real crystal the potential function is more complicated than the Kronig-Penney squares. Thus, the energy gaps are by no means uniform in size. The gap sizes may be changed by the introduction of impurities or imperfections of the lattice.

- These facts concerning the energy gaps are of paramount importance in understanding the electronic behavior of semiconductors.
Semiconductors Physics:

### Band Theory and Conductivity

- Band theory helps us understand what makes a conductor, insulator, or semiconductor.
  1. Good conductors like copper can be understood using the free electron
  2. It is also possible to make a conductor using a material with its highest band filled, in which case no electron in that band can be considered free.
  3. If this filled band overlaps with the next higher band, however (so that effectively there is no gap between these two bands) then an applied electric field can make an electron from the filled band jump to the higher level.
- This allows conduction to take place, although typically with slightly higher resistance than in normal metals. Such materials are known as semimetals.

### Valence and Conduction Bands

- The band structures of insulators and semiconductors resemble each other qualitatively. Normally there exists in both insulators and semiconductors a filled energy band (referred to as the valence band) separated from the next higher band (referred to as the conduction band) by an energy gap.
- If this gap is at least several electron volts, the material is an insulator. It is too difficult for an applied field to overcome that large an energy gap, and thermal excitations lack the energy to promote sufficient numbers of electrons to the conduction band.

### Smaller energy gaps create semiconductors

- For energy gaps smaller than about 1 electron volt, it is possible for enough electrons to be excited thermally into the conduction band, so that an applied electric field can produce a modest current.

The result is a semiconductor.

### Temperature and Resistivity

- When the temperature is increased from $T = 0$, more and more atoms are found in excited states.
- The increased number of electrons in excited states explains the temperature dependence of the resistivity of semiconductors.

Only those electrons that have jumped from the valence band to the conduction band are available to participate in the conduction process in a semiconductor. More and more electrons are found in the conduction band as the temperature is increased, and the resistivity of the semiconductor therefore decreases.

### 11.2: Semiconductor Theory

- At $T = 0$ we expect all of the atoms in a solid to be in the ground state. The distribution of electrons (fermions) at the various energy levels is governed by the Fermi-Dirac distribution of Equation (9.34):

$$F_{FD} = \frac{1}{\exp(\beta(E - E_F)) + 1}$$

$\beta = (kT)^{-1}$ and $E_F$ is the Fermi energy.

### Some Observations

- Although it is not possible to use the Fermi-Dirac factor to derive an exact expression for the resistivity of a semiconductor as a function of temperature, some observations follow:
  1. The energy $E$ in the exponential factor makes it clear why the band gap is so crucial. An increase in the band gap by a factor of 10 (say from 1 eV to 10 eV) will, for a given temperature, increase the value of $\exp(\beta E)$ by a factor of $\exp(9\beta E)$.
    - This generally makes the factor $F_{FD}$ so small that the material has to be an insulator.
  2. Based on this analysis, the resistance of a semiconductor is expected to decrease exponentially with increasing temperature.
    - This is approximately true—although not exactly, because the function $F_{FD}$ is not a simple exponential, and because the band gap does vary somewhat with temperature.
A useful empirical expression developed by Clement and Quinnell for the temperature variation of standard carbon resistors is given by

\[
\log R + \frac{K}{\log R} = A + \frac{B}{T}
\]

where \( A, B, \) and \( K \) are constants.

When electrons move into the conduction band, they leave behind vacancies in the valence band. These vacancies are called holes. Because holes represent the absence of negative charges, it is useful to think of them as positive charges.

Whereas the electrons move in a direction opposite to the applied electric field, the holes move in the direction of the electric field.

A semiconductor in which there is a balance between the number of electrons in the conduction band and the number of holes in the valence band is called an intrinsic semiconductor.

Examples of intrinsic semiconductors include pure carbon and germanium.

The addition of arsenic to silicon creates what is known as an \( n \)-type semiconductor \((n\text{ for negative})\), because it is the electrons close to the conduction band that will eventually carry electrical current.

The new arsenic energy levels just below the conduction band are called donor levels because an electron there is easily donated to the conduction band.

It is possible to fine-tune a semiconductor’s properties by adding a small amount of another material, called a dopant, to the semiconductor, creating what is called an impurity semiconductor.

As an example, silicon has four electrons in its outermost shell (this corresponds to the valence band) and arsenic has five.

Thus while four of arsenic’s outer-shell electrons participate in covalent bonding with its nearest neighbors (just as another silicon atom would), the fifth electron is very weakly bound.

It takes only about 0.05 eV to move this extra electron into the conduction band.

The effect is that adding only a small amount of arsenic to silicon greatly increases the electrical conductivity.

Consider what happens when indium is added to silicon.

- Indium has one less electron in its outer shell than silicon. The result is one extra hole per indium atom. The existence of these holes creates extra energy levels just above the valence band, because it takes relatively little energy to move another electron into a hole.

- Those new indium levels are called acceptor levels because they can easily accept an electron from the valence band. Again, the result is an increased flow of current (or, equivalently, lower electrical resistance) as the electrons move to fill holes under an applied electric field.

- It is always easier to think in terms of the flow of positive charges (holes) in the direction of the applied field, so we call this a \( p \)-type semiconductor \((p\text{ for positive})\).

- acceptor levels \( p \)-Type semiconductors

- In addition to intrinsic and impurity semiconductors, there are many compound semiconductors, which consist of equal numbers of two kinds of atoms.
Thermoelectric Effect
- In one dimension the induced electric field $E$ in a semiconductor is proportional to the temperature gradient, so that

$$E = Q(\frac{dT}{dx})$$

where $Q$ is called the thermoelectric power.

- The direction of the induced field depends on whether the semiconductor is $p$-type or $n$-type, so the thermoelectric effect can be used to determine the extent to which $n$- or $p$-type carriers dominate in a complex system.

Thermoelectric Effect
- When there is a temperature gradient in a thermoelectric material, an electric field appears.
  - This happens in a pure metal since we can assume the system acts as a gas of free electrons.
  - As in an ideal gas, the density of free electrons is greater at the colder end of the wire, and therefore the electrical potential should be higher at the warmer end and lower at the colder end.
  - The free-electron model is not valid for semiconductors; nevertheless, the conducting properties of a semiconductor are temperature dependent, as we have seen, and therefore it is reasonable to believe that semiconductors should exhibit a thermoelectric effect.
  - This thermoelectric effect is sometimes called the Seebeck effect.

Formation of Energy Bands
From quantum mechanics, we know that the energy of the bound electron of the hydrogen atom is quantized with associated radial probability density functions.

When two hydrogen atoms are brought in close proximity, their wave functions will overlap, which means the two electrons will interact. This interaction results in the quantized energy level splitting into two discrete energy levels.

According to the Pauli exclusion principle, the total number of quantum states will remain the same after the joining of atoms to form a system (crystal).

There will be many energy levels within the allowed band in order to accommodate all of the electrons in a crystal. As an example, suppose that we have a system of $10^{19}$ one-electron atoms and the width of the energy band at the equilibrium inter-atomic distance is 1 eV. If the spacing between neighboring energy levels is the same, the difference in neighboring energy levels will be $10^{-19}$ eV, which is extremely small so that we have a quasi-continuous energy distribution through the band.
Example: consider an electron traveling at a velocity of 107 cm/sec, if the velocity increases by 1 cm/sec, calculate the change in its kinetic energy.

Solution:
\[
\Delta E = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 = \frac{1}{2} m (v_f^2 - v_i^2)
\]
\[
v_f = v_i + \Delta v
\]
\[
\Delta v \ll v_i
\]
\[
v_f^2 = (v_i + \Delta v)^2 = v_i^2 + 2v_i \Delta v + (\Delta v)^2 = v_i^2 + 2v_i \Delta v
\]
\[
\Delta E = \frac{1}{2} m (2v_i \Delta v) = m v_i \Delta v
\]
\[
\Delta E = (9.11 \times 10^{-38} \text{ J}) (0.01) = 9.11 \times 10^{-39} \text{ J}
\]
\[
E = \frac{9.11 \times 10^{-39}}{1.6 \times 10^{-19}} \approx 5.7 \times 10^{-\text{eV}}
\]

Comment: the kinetic energy change is orders of magnitude larger than the energy spacing in the allowed energy band, which suggests that the discrete energies within an allowed energy band can be treated as a quasi-continuous distribution.

Allowed and Forbidden Energy Bands

Consider again a periodic arrangement of atoms. Each atom contains electrons up to the \(n = 3\) energy level. If these atoms are brought together, the outermost electrons in the \(n = 3\) energy shell will begin to interact and split into a band of allowed energies. As the atoms move closer, the electrons in the \(n = 2\) shell, and finally the innermost electrons in the \(n = 1\) shell, will also form two bands of allowed energies.

Formation of energy bands - diamond lattice crystal

The energy-momentum curve of a free electron

The effective mass \(m_e\) and the \(E-p\) curve

For silicon, we need only consider the \(n = 3\) level because the first two energy shells are completely full and are tightly bound to the nucleus.

The 3s state contains two quantum states per atom, and the 3p state contains six quantum states per atom.

At the equilibrium inter-atomic distance, the 3s and 3p bands mix together and split to form two new bands with four quantum states per atom in the lower band and four quantum states per atom in the upper band. At absolute zero degrees, all the states in the lower band (the valence band) will be occupied by electrons and thus are full. All the states in the upper band (the conduction band) will be empty. The energy difference between the top of the valence band and the bottom of the conduction band is the bandgap energy.
Energy band structures of Si and GaAs. Circles (•) indicate holes in the valence bands and dots (◦) indicate electrons in the conduction bands.

Intrinsic Carrier Densities – derivation of "density of state"

\[ n(E) = N(E)F(E) \]

- derivation of \( N(E) \)
- \( \frac{E}{\lambda} = \frac{\text{vol}}{\lambda} \) and \( \lambda = \frac{h}{p} \)
- \( E_p = \text{vol} \Rightarrow \text{vol} \frac{dE}{d\text{vol}} = \frac{h}{p} \)
- \( E = \frac{E_p}{2m} \Rightarrow \text{vol} \frac{dE}{d\text{vol}} = \frac{1}{2} (2m)^{1/2} E^{3/2} \)
- \( N(E) = 4\pi \left( \frac{2m}{\hbar^2} \right)^{1/2} E^{3/2} \)

Intrinsic carrier densities in Si and GaAs as a function of the reciprocal of temperature.

Energy band of different material

(a) a conductor
(b) a semiconductor
(c) an insulator

Intrinsic Carrier Densities

- \( n(E) = N(E)F(E) \)
- \( p(E) = P(E)F(E) \)

(a) Intrinsic semiconductor. (b) Schematic band diagram. (c) Density of states. (d) Carrier concentration.

Extrinsic Carrier from Dopant

- n-type Si with donor (arsenic)
- p-type Si with acceptor (boron)
Semiconductors Physics:

Extrinsic Carrier Densities

Fermi level for Si and GaAs as a function of temperature and impurity concentration. The dependence of the bandgap on temperature.

Electron Density v.s. Temperature

The Energy Band and the Bond Model

The performance of many semiconductor devices is related to the current-voltage (I-V) characteristics. We will need to consider electrical conduction in solids, which is related to the band theory.

We consider silicon. At $T = 0$ K, each silicon atom is surrounded by eight valence electrons that are in their lowest energy state. In terms of the band theory, the 4N states in the valence band are completely occupied by electrons, and the conduction band is completely empty.

The Energy Band and the Bond Model

At $T > 0$ K, a few electrons may gain enough thermal energy to break the covalent bond and jump into the conduction band.

As a negatively charged electron breaks away from its covalent bonding position, a positively charged "empty state" is created in the original covalent bonding position in the valence band. As $T$ further increases, more electrons jump into the conduction band and more positive "empty states" are created in the valence band.

The Energy Band and the Bond Model

This bond breaking can also be related to the $E$ versus $k$ energy bands.

At $T = 0$ K, the valence band is completely full and the conduction band is completely empty. At $T > 0$ K, some electrons have gained enough energy to jump to the conduction band and have left empty states in the valence band.

We assume at this point that there is no external forces. The electron and "empty state" distributions are symmetrical with $k$. 

ABV-IIITM-Gwalior (MP) India
Drift Current

If we have a collection of positively charged ions with a volume density \( N \) (cm\(^{-3}\)) and an average drift velocity \( \nu_d \) (cm/sec), then the drift current density is

\[ J = qN \nu_d \text{(A/cm}^2\text{)} \]

If we consider the individual ion velocities with \( \nu_i \) being the velocity of the \( P \) ion and take the summation over a unit volume, then we have

\[ J = q \sum \nu_i \]

Electrons are negatively charged. A net drift of electrons in the conduction band will give a current. However, the electron distribution in the conduction band is an even function of \( \nu \). Recall that \( \nu = \frac{h}{m} \) for a free electron is related to the momentum. The number of the electrons with \( \nu = \frac{h}{m} \) value is equal to that with \( -\frac{h}{m} \) value. The net drift current density due to these electrons is zero.

Electron Effective Mass

The movement of an electron in a crystal lattice will, in general, be different from that of an electron in free space. In addition to an externally applied force, there are internal forces in the crystal due to positively charged ions and negatively charged electrons.

\[ F_{\text{net}} = F_{\text{ext}} + F_{\text{int}} = ma \]

We use an effective mass \( m^* \) to take into account both the electron mass and the effect of the internal forces.

\[ F_{\text{int}} = m^*a \]

We can also relate the effective mass of an electron in a crystal to the \( E \) versus \( k \) curves. Consider the case of a free electron:

\[ \frac{dE}{dk} = \frac{h^2 k}{m} \]

Apply the result from the Kronig-Penney model to the electron in the bottom of an energy band. The energy near the bottom of the energy band may be approximated by a parabola:

\[ (E - E_0) = C_1(k - k_0)^2 \]

\[ \frac{dE}{dk} = 2C_1 \]

If valence electrons gain a certain amount of energy, they can hop into the “empty state.” The movement of a valence electron into the “empty state” is equivalent to the movement of the positively charged “empty state” itself.

This charge carrier is called a hole and can also be thought of as a classical particle whose motion can be modeled using Newtonian mechanics.

Concept of the Hole

If valence electrons gain a certain amount of energy, they can hop into the “empty state.” The movement of a valence electron into the “empty state” is equivalent to the movement of the positively charged “empty state” itself.

This charge carrier is called a hole and can also be thought of as a classical particle whose motion can be modeled using Newtonian mechanics.
**Concept of the Hole**

Consider the electrons in the valence band and their drift current density:

\[ J = -e \sum_{\text{valence}} v_e \]

Remember that we have:

\[ \nu(E) = \frac{1}{h} \left( \frac{dE}{dk} \right) \]

The band is symmetric with \( k \). For every electron with a velocity \( v \), there is a corresponding electron with a velocity \( -v \). Because the band is full, the distribution of electrons with respect to \( k \) cannot be changed by an external force. Then we have

\[ -e \sum_{\text{valence}} v_e = 0 \]

\[ J = +e \sum_{\text{empty}} v_e \]

The \( v_e \) in the summation is associated with the empty state.

---

**Hole Effective Mass**

If we consider an electron near the top of a valence band and use Newton’s force equation for an applied electric field, we will have

\[ F = ma = -eE \]

\[ a = -\frac{eE}{m} \]

\[ a = \frac{eE}{m_e^*} \]

The motion of electrons in a nearly full band can be described by considering just the empty states. The band can be modeled as having particles with a positive electronic charge and a positive effective mass. The density of these particles in the valence band is the same as the density of empty electronic energy states. This new particle is called the hole. The hole has a positive electronic charge and a positive effective mass denoted by \( m_e^* \). This quantity is used to relate quantum mechanics to classical mechanics.

---

**Metals, Insulators, and Semiconductors in terms of Energy Band Structures**

A band with relatively few electrons. If an electric field is applied, the electrons can move to higher energy states to generate a current.

An almost full band. We can consider the holes, which move under an electric field to generate a current.

A representative energy band for a semiconductor at \( T > 0 \) K. The bandgap energy of a semiconductor is on the order of 1 eV. The conductivity of a semiconductor can be varied over many orders of magnitude.

---

**Concept of the Hole**

It is equivalent to placing positively charged particles in the empty states and assuming all other states in the valence band are empty, or neutral charged.

Consider an electron near the top of a valence band:

\[ (E - E_0) = -C_1 (k - k_0)^2 \]

\[ \frac{d^2 E}{dk^2} = -2C_1 \]

\[ \frac{1}{h^2} \frac{d^2 E}{dk^2} = -2C_1 \cdot \frac{1}{h^2} = m \]

\( C_1 \) is a positive quantity and thus \( m^* \) is a negative quantity.

---

**Metals, Insulators, and Semiconductors in terms of Energy Band Structures**

There will be no current for a completely empty band because there are no particles to move.

There will also be no current for a completely full band because of the band symmetry with respect to \( k \).

Insulators have energy bands either completely filled or completely empty and bandgap energies in the range of 3.5 to 6 eV or larger. For comparison, the \( \text{EF} \) at room temperature (25 °C) is 25.7 meV.

---

Metals have low resistivity values. The energy band diagram for a metal can be in one of two forms.

One case is a partially filled band in which there are many electrons available for conduction.

The other case is that the conduction band overlaps with the valence band. There are a large number of electrons as well as a large number of holes for conduction.
Simplified schematic drawing of the Czochralski puller. Clockwise (CW), counterclockwise (CCW).

Energy band diagrams are symmetric in \( E \) so that only the positive axis is displayed. GaAs is a direct bandgap semiconductor. Si is an indirect bandgap semiconductor. The electron transitions in indirect bandgap semiconductors must involve changes of crystal momentum, and thus the emission efficiency of indirect bandgap semiconductors is much smaller than that of direct bandgap semiconductors. The curvature is related to the effective masses of electrons and holes, which are different along different directions. Usually a statistical average is used for device calculations.